2020

23rd Annual High School Mathematical Contest in Modeling (HiMCM)
Summary Sheet

Team Control Number: 10550

Problem Chosen: A

Many high school students want to seek appropriate summer jobs during summer vacations. Doing summer jobs can benefit the students from many aspects. Not only the students can earn money from summer jobs, they can learn skills and new knowledge, gain work experience and know new friends. High school students have already had certain skills and can do suitable jobs. While many summer jobs and positions prevail on online job websites, many high school students find themselves hard to find suitable jobs.

We developed an analytic network process (ANP) model to solve best summer job selection problem. This problem in nature is a multiple criteria decision making problem. It involves many decision factors and cannot be settled unless the factors, job properties, and applicant situation and preferences are comprehensively considered. In our solution, summer jobs are evaluated from total 15 criteria in four aspects: salary, benefit, work conditions, and job suitability. Salary is the most important factor affecting job choice. Benefit refers to the improvement of students' soft power, such as technical ability, communication ability, leadership and friendship. Working conditions include working place and environment, working hours, flexibility of work policy. Suitability refers to the degree of matching between job requirements and students' skills and interests. Both job properties and user situation/preferences are fully considered in the model. Therefore, this model can recommend best jobs in alignment with users' situation/preferences.

The model is validated using ten jobs and ten fictional people and works as expected. Not only the best job, which has a highest score of priority, can be recommended, but all candidate jobs are ranked according to their computed priorities, providing more job options for users. A mobile App can be developed to present this solution to high school students as potential users. With this App, high school student users are required to provide a few information and let us know their preferences on choosing job, and the established solution can do the rest and return the ranked jobs to the users.

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Solution for Problem A of HiMCM 2020

1 Introduction

1.1 Background

In summer vacations, many high school students want to seek appropriate summer jobs. Having a job in high school can not only be a great experience, it can also set you up to get even better jobs in college and beyond. There are many benefits to getting summer jobs for high school students. The students can earn some money, learn extra skills and knowledge, and gain work experience that would be helpful for major choice in university in the future. By doing summer jobs, the students may learn more about what kind of career they want in future. There are usually many summer jobs available around the students' residence. Some involve physical activities, such as cashier at a store, lifeguard in swimming pool, waiter at a restaurant and so on, while others are mostly sedentary and need some professional skills. Those office jobs suitable for high school students may include programmer, data analyst, office administrator, research assistant, zoo assistant, and writing tutors. Different students have different situation and preferences. Some may have excellent technical skills, while others may possess excellent soft skills (e.g. communication skills, leadership). Some may expect good salary, while others may just want to improve themselves even if no salary is paid. While many summer jobs and positions overwhelm on online job websites, it is hard for students to find best jobs. We mean a best job for high school students as a job offering great gains, matching students' preferences, able to be completed by high school students, and of course, having great chance for the student to own it.

In this article, we aim to develop a mathematical model to help high school students get their best jobs, by considering students' own capabilities, preferences and expectations, as well as salaries the jobs can offer and job suitability.

1.2 Problem Restatement

Question 1: What factors should be considered when high school students are looking for a summer job? What benefits can students get by doing the summer job? At the same time, what skill requirements are needed for the summer job? These factors may be quantitative or qualitative, constant or variable, and deterministic or probabilistic.

Question 2: Develop a model to evaluate the summer job options and give a rank. The factors, the properties of summer jobs, and the situation and preferences of high school student should both be considered as the basis of evaluation. The high students can use the model results to choose their "best job".

Question 3: Create at least ten fictional persons with reasonable data. What data do you choose to create these persons and why? Test the model developed in the second question and analyze the results of the application of the model on these persons.

Question 4: How would you present your model for a person to understand and use?

A webpage, an app or a school newspaper article? Describe and provide the layout of your proposed presentation.

2 Problem analysis

Question 1: We define a best summer job for high school students as a job the student can have most gains (financial, intellectual, and mental, etc.), most likes (mostly match his/her preferences), and has great opportunity to own it. The benefits should not be limited to high pay, but also benefits of all aspects. We will develop criteria including salary, the benefits in terms of improvements in technical skills, communication skills, leadership/teamwork, and skills related to future university major or career, and knowing new friends. Thus, the factors to determine whether it is a best summer job should include not only the gains from the job, but also the chance of getting this job and how the work conditions are. To make a full evaluation, job characteristics/properties and user situation/preferences should be included as evaluating factors. The skill requirements depend on the jobs to be applied. Different jobs have different skill requirements. To simplify this situation, we identify some common job types with which the skill requirements can be pre-identified. We can collect common summer jobs from major job websites, and then develop the factors and identify most common skill requirements. Students' situation/preferences can be collected via interactive forms and questionnaire. Combining these two aspects, a set of comprehensive and reasonable factors that influence the job selection can be developed.

Question 2: The best job selection problem in nature is a multiple criteria decision making (MCMD) problems. We need to establish a decision-making model, which can take the "influencing factors" obtained in question 1 as the evaluation criteria. The model can determine if the job is best for the users by considering all criteria including gains, users' situation and preferences. The better the matching degree, the higher the score, the better the job. As this is a very complex problem involving multiple decision factors that might interrelated with each other and many alternative jobs, the model should be selected appropriately and be able to deal with this complex problem.

Question 3: We need to set up ten fictional persons representative of some aspects. Each person should have his/her own characteristics, which are associated with the influencing factors mentioned in questions 1 and 2. As we cannot collect real persons' information, we can use some random approach to generate fictional persons with randomly assigned profiling information. Students' situation and preferences are then created and input to the model for solving best jobs. The resulting best jobs can be analyzed by referring to person's information.

Question 4: We can design a mobile App. The job database and the model can install over cloud, and the App offers the interfaces to receive user information and transfers them to the model running remotely. The App provides functions such user information collection, start or stop the model running and display the results. The screenshots of the App will present in this article.

3 Assumptions

Assumption 1: A best summer job for high school students is defined as a job the student can have most gains (financial, intellectual, and mental, etc.), most likes (mostly match his/her preferences), and has great opportunity to own it.

Justification: A job with very high salary may be someone's best job, but not all's. Especially for high school students, pursuing high pay is not unique purpose in looking for a job. Sometimes a student prefers learning knowledge or improving skills from a job and the money earned is second. Therefore, matching the students' preferences are very important to define a best job. The chance for a student to get this job is another important criterion to define a best job. In this sense, a best job is also a most suitable job considering the student own's situation.

Assumption 2: Many direct or indirectly costs incurred during seeking a summer job are ignored and the jobs can be characterized by a limited number of factors.

Justification: Model is a simplified representation. In the model, we can only pay attention to major points and ignore minor ones. As summer jobs are usually temporal jobs, the costs relating to seeking summer jobs are not big and thus ignored. Although they are both called camp counselor, counselor jobs for elementary and junior school students require different skills and offer different salaries. In our testing job data, they are simplified. All counselor jobs are considered the same. In theory our model supports very detailed profiling of jobs and users.

Assumption 3: Randomly generated person information can represent real people situation and preferences, and data necessary for the model can be obtained.

Justification: Each person has his/her own preferences and opinions on doing something. To represent real user cases, a better way is to collect a large number of real person's data. However, we are unable to use real person's data and turns to create fictional persons as requested whose basic information and preferences are randomly generated. We use very limited person data to test our model. Job data can be collected from online job website all over the world. To collect and process those data involve many techniques and some data might be copyrighted. We assume those data can be obtained because of our main purpose is to develop a model. The model is then tested with a small number of real job records acquired from a job website and fictional people.

4 Methodology

4.1 Overall work flow

Our method consists of three steps (Figure 1): Job data collection and preprocessing, user information collection, and analytic network process (ANP) solver.

- Job data collection and preprocessing: job data are obtained from online job websites. After certain quality control, some filters are applied to exclude jobs that are obviously unsuitable for high school students. The raw unstructured data are processed into structured data to provide support for ANP computation.
- User information collection: user situation and preferences are collected through

interactive forms and questionnaire respectively. Student situation/profile includes his/her location, skills, interests, personality types, expected salary and recreation hours. Preference information includes the user's preferences towards established evaluation criteria and jobs types.

- ANP Solver: We established 4 clusters and 15 criteria to form the ANP super matrix to solve job's priority considering users' preferences. Job data and user information are needed to estimate weight sub-matrices. Consistency are then checked and limiting matrix is reached after repeatedly multiplying the weighted super-matrix with itself. Jobs' priorities are then given.

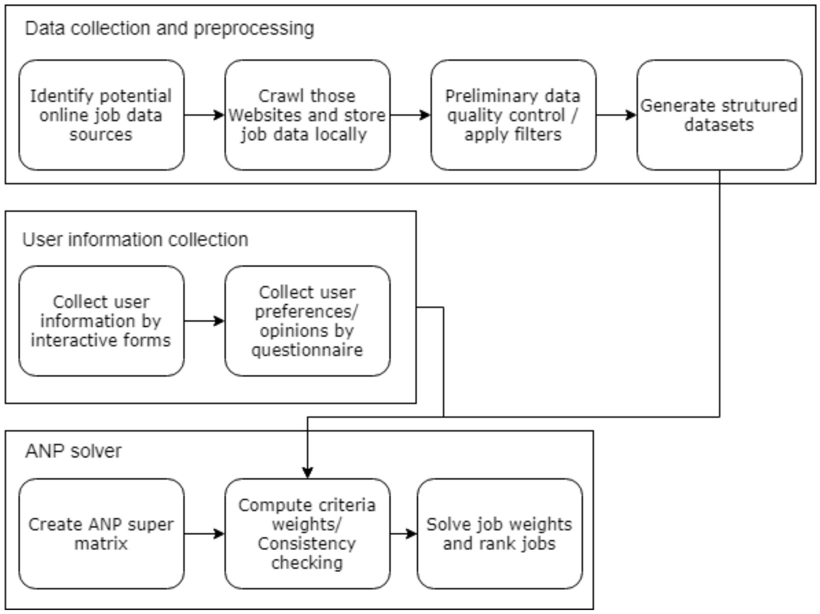

Figure 1 Methodological flowchart

4.2 Job and user data collection and preprocessing

Job data is obtained from online job websites. Many job websites are available in various countries, such as in UK, studentjob.co.uk provides many summer jobs for students; in China, the data of summer jobs can be obtained from 58.com. In addition, goodwall.io, summerjobs.com and indeed.com all provide a lot of jobs. Considering the particularity of high school students, we limit candidate jobs to the same city where students are located.

We use Web crawlers (such as well-known scrapy, pyspider, etc.) to grab job data from these online work websites, but the captured data is unstructured. Before entering our model, we need to preprocess and structure these raw data. The preprocessing includes removal of duplicate records, and preliminary data quality control, etc. An example of structured job data is shown in Appendix A. Those job data are from https://www.studentjob.co.uk. Each job has a specific workplace. If a job has multiple workplaces, multiple jobs are created in line with every workplace. This is because we need to calculate the distance between the user and the workplace to determine commuting cost, which will affect final job selection.

User situation and preferences data are obtained through interactive forms and questionnaires. According to the obtained user data, further filtering is applied to exclude obviously unsuitable jobs. For example, excluded are jobs with salary that is far lower than expected salary or requiring skills that the student does not have. The jobs that enter our model are therefore limited to a subset of jobs that satisfy the basic conditions. The questionnaire is used to obtain the user's opinions and preferences for evaluating jobs, which are necessary to calculate the weights of evaluation criteria. For example, students are prompted to answer these questions to determine their opinions towards the importance of the four ANP clusters in evaluating goodness of a job:

> Please score the importance (0-100) of the following items when you are going to choose a summer job, with the most important one having a score of 100:
>
> a) payment____;
>
> b) technical and social skills improvement____;
>
> c) work conditions____;
>
> d) degree of interest and skill matches____.

4.3 Mathematical model

4.3.1 Analytic network process

ANP is a complex tool to deal with multi-objective decision-making problems. It uses a network structure to consider the dependence between models and the interaction and feedback between evaluation criteria (Saaty, 1990). Therefore, ANP has the advantages that other decision-making methods, such as AHP and decision tree, do not have. Moreover, in making decision for best jobs, we need to consider that the various criteria may affect each other, and user situation/preferences and job information jointly affect jobs priorities. The mathematical model based on ANP can meet these needs.

ANP network is expressed as nodes and arcs. Nodes can express criteria or alternatives and arcs can express the relationship between nodes. Arc may have two directions, meaning "depend on" and "has influences on" respectively. Nodes in ANP can be classified as multiple clusters, including goal, criteria and alternative clusters. Criteria and alternative clusters can include many nodes representing specific criteria and alternatives respectively. In our problem, alternatives are different jobs. When one or more nodes in a cluster are associated with one or more nodes in another cluster, a connection is established between the two clusters.

Our problem is constructed as an ANP network as shown in Figure 2. The Best Summer Job node represents a goal cluster, which only includes one node and is associated with four criteria clusters, namely salary, benefit, work conditions, and sustainability. Each criteria cluster includes a different number of criteria nodes, such as salary cluster only includes one salary node while benefit cluster includes 5 nodes. There are many nodes in the alternatives cluster, which represent different jobs respectively.

The blue arcs in Figure 2 indicate cluster level associations, and the black lines indicate node level associations. Each arc has at most two directions of association. For example, the associations from Job1 to work conditions criteria indicate relative weights of work conditions criteria influencing Job1; the associations from the workplace environment criteria to all candidate jobs represent relative weights of jobs on this criterion. These weights are calculated by establishing a comparison matrix. The bidirectional associations between jobs and criteria indicate the roles and feedbacks of job characteristics on criteria.

4.3.2 Criteria for evaluating candidate jobs

We measure whether a job is best for the user from four aspects:

- *Salary* indicates the money we have made. A job with high salary will attract a lot of people. Considering that some people attach great importance to salary, we separate it from other benefits.
- *Benefit* refers to the improvement of one's own ability by completing the work, including the improvement of technical and social skills. A job with low salary but a lot of self-improvement can also attract many high school students, especially when the working experience can enrich the resume, or help apply universities in the future.
- *Work conditions* indicates the working conditions of the job, including work environment, working hours, job flexibility, and the convenience of commuting etc. These factors will affect the user's judgment of whether a job is good or bad.
- *Suitability* indicates the fitness between individual user and candidate jobs. If a job is consistent with our interests or our personality, we may lower our expectations in other aspects. At the same time, a job may also have skill requirements. The more students meet these requirements, the higher the probability of being hired.

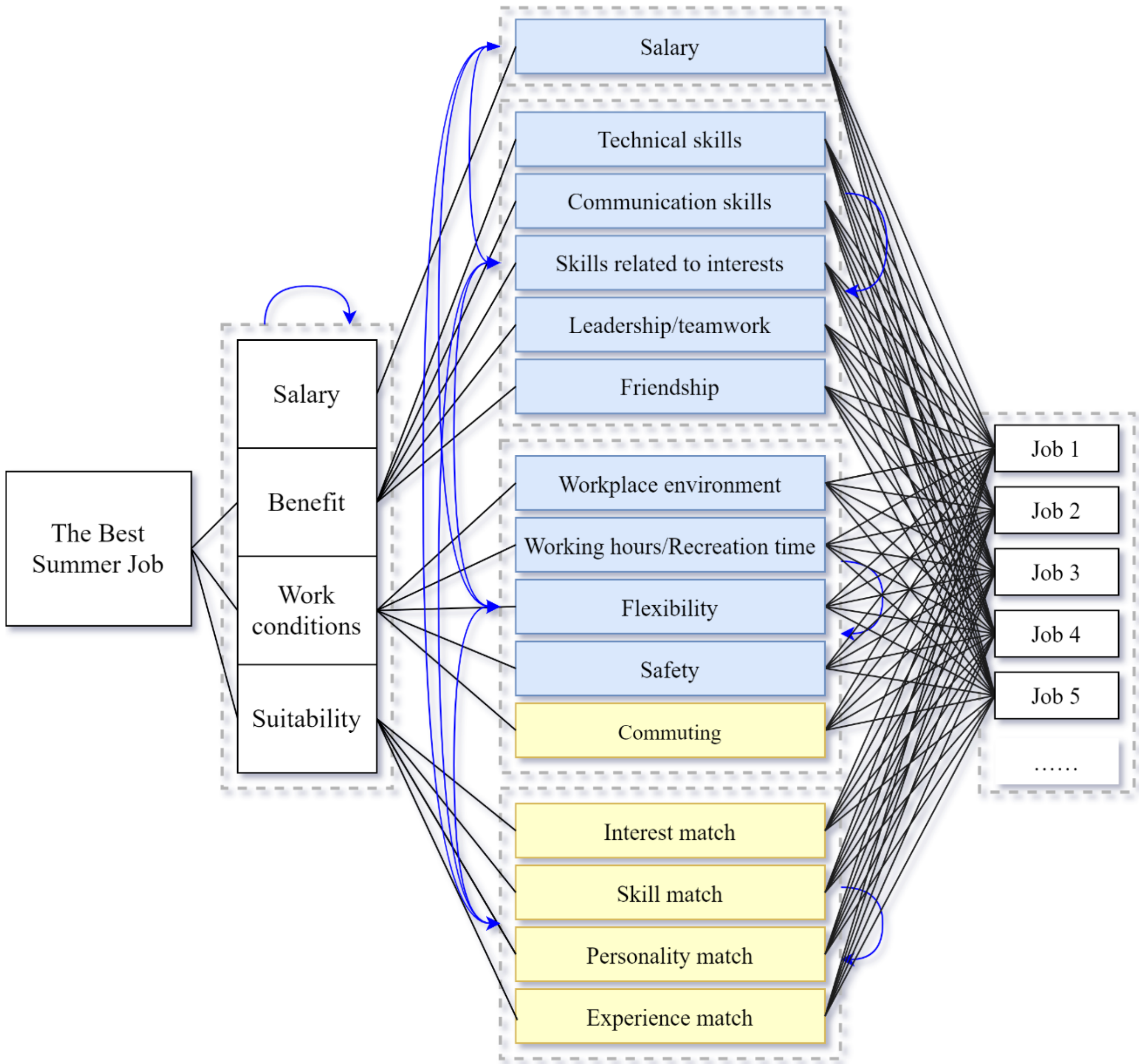

Figure 2 Network structure in the ANP model

The four first level criteria are represented as clusters in the ANP, and the second level criteria as shown in Figure 2, include 15 criteria in total. The criteria in blue indicate that their weights only depend on the job itself, while the weights of yellow-colored criteria also depend on students' situation/preferences. Meanwhile, for different student users, the four clusters are also given different weights to reflect their distinguished preferences. The criteria clusters and criteria are shown in Table 1.

Table 1 Established criteria for best job selection. Codes are included within parentheses.

1st criteria	2nd criteria
Salary (S)	Salary (S)
Benefit (B)	Technical skills (B1)
	Communication skills (B2)
	Skills related to interests (B3)
	Leadership/teamwork (B4)

	Friendship (B5)
Work conditions (W)	Workplace environment (W1)
	Working hours/recreation time (W2)
	Flexibility (W3)
	Safety (W4)
	Commuting (W5)
Suitability (U)	Interest match (U1)
	Skill match (U2)
	Personality match (U3)
	Experience match (U4)

4.3.3 ANP super matrix and weight determination

Based on goal node, criteria nodes and job nodes, we establish the ANP super matrix (Table 2). W_{mn} represents how the *m*-th cluster (row) affects the *n*-th cluster (column). The salary cluster has only one node, so the elements of W_{21} and W_{26} are all taken as 1.0. The values of the column where the goal cluster is located does not affect the limiting matrix (Kadoić, 2018), so W_{i1}(i=3,4,5) is set to zero. The remaining 16 sub-matrices need to be determined by paired comparison on node level. Each sub-matrix has a sum of each column of 1. Therefore, the resulting super matrix is unweighted and needed to be normalized prior to computing for the limiting matrix.

The codes of criteria in the table are consistent with Table 1. “Best job” stands for the goal cluster as numbered 1; “Jobs” represents the alternatives cluster as numbered 6. We need to determine 21 weight sub-matrices (denoted as W in Table 2).

The sixteen sub-matrices with unknown weights to be solved can be divided into several categories according to the association:

- W_{i6} and W_{6j}(i=3,4; j=2,3,4) reflect the associations and feedbacks between individual jobs and criteria, and they are calculated with job information as input. The exceptions are the weights related to the commuting criterion, which require more information such as location of user and commuting means to the workplace in addition to job information;
- W_{56} and W_{65} are weight matrices representing the joint influences of user situation/preferences and job characteristics. both user and job information are needed in the calculation;
- The remaining weights sub-matrices (W_{42}, W_{52}, W_{33}, W_{43}, W_{53}, W_{44}, W_{54}, W_{45}, and W_{55}) reflect the interactions and feedbacks between and within the criteria clusters, and these weights reflect general understanding of those evaluation criteria, independent of individual user and independent of job. In our calculations it is determined by our experience.

Table 2 The ANP supermatrix for best job selection

			S	B					W					U				Jobs			
		Best job	**S**	**B1**	**B2**	**B3**	**B4**	**B5**	**W1**	**W2**	**W3**	**W4**	**W5**	**U1**	**U2**	**U3**	**U4**	**job 1**	**job 2**	**job 3**	......
	Best job	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
S	**S**	W_{21}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	W_{26}			
B	**B1**	W_{31}	0	W_{33}					0	0	0	0	0	0	0	0	0	W_{36}			
	B2		0						0	0	0	0	0	0	0	0	0				
	B3		0						0	0	0	0	0	0	0	0	0				
	B4		0						0	0	0	0	0	0	0	0	0				
	B5		0						0	0	0	0	0	0	0	0	0				
W	**W1**	W_{41}	W_{42}	W_{43}					W_{44}					W_{45}				W_{46}			
	W2																				
	W3																				
	W4																				
	W5																				
U	**U1**	W_{51}	W_{52}	W_{53}					W_{54}					W_{55}				W_{56}			
	U2																				
	U3																				
	U4																				
Jobs	**job 1**	**P**	W_{62}	W_{63}					W_{64}					W_{65}				0	0	0	0
	job 2																	0	0	0	0
	job 3																	0	0	0	0
	......																	0	0	0	0

We use the fundamental scale of absolute numbers (Saaty,2008) which are established to make paired comparisons. The scale has nine different intensities: 1 means that two elements in a pair are equally important with respect to the higher level element; 9 means extreme importance of one element over another. Values of the elements in the comparison matrix usually indicate relative importance between two criteria. In our problem, the criteria variables include both qualitative and quantitative types. For example, we can only rank relative importance of each criterion in the Benefit cluster with respect to a certain job. But salary as a criterion can be calculated according to each job, and in many cases, the commuting cost can be calculated according to the locations of students and jobs they are applying. When these cases appear, the absolute values of the rates between these criteria can be used to determine the importance.

When the weights are determined by the comparison matrix, job-related weights are derived from structured job data, and user-related weights are obtained from the collected user data via interactive forms and specially designed questionnaire. In order to reduce the computational costs, we identify some commonly used types of jobs, such as Camp Counselor, Store Cashier and so on. Thus, many job-related weights can be derived from job types instead of using individual job details. For example, Camp Counselor type jobs benefit the students more in social skill development and building leadership than any Store Cashier job. The working location of Camp counselor is usually outside, while Store Cashier’s working place is in the interior. Job-related information is thus stratified into two layers, one based on the job type and the other related to individual job details. Those details should be carefully collected. For example, the same type of "Store cahier" jobs can differ in salary they can offer.

Because the calculation of sub-matrices is complicated, we developed the following strategies. When jobs are updated, we only need to recalculate the sub-matrices related to individual job details; when new user uses our model, we only need to recalculate sub-matrices related to user. The rest of the weights do not need to be updated. Appendix B demonstrates an unweighted supermatrix including 10 jobs as listed in Appendix A that takes into account a fictional student's situation and preferences.

Let's take W_{65}, which considers both user and job, as an example to introduce a general procedure to determine a weight sub-matrix. In the user data collection stage, we collected the user's basic information, including the student's location, skills, interests and personality types, etc. These information in combination with the structured job data are used to evaluate the importance of relevant criteria through a paired comparison method. After the comparison matrix gets constructed, the eigenvector is calculated to obtain weights.

In a supermatrix built for a student user, the column in the W_{65} sub-matrix related to the criteria of interest match is solved as follows: firstly, as per the user's profile information, he/she pays more attention to the improvement of his/her own skills and abilities; he/she is outgoing and enthusiastic with some teamwork skill and leadership; and he/she has some experience in student work. Those user information are then compared to the relevant information stored in the structured job dataset (Appendix A) to evaluate the degrees of interest match between the user and 10 jobs. As the user is warm-hearted and has good teamwork skills and leadership, and hopes to further improve his/her ability, the jobs such as Languages Teachers (online), Student Brand Ambassador, and Summer Resident Camp Counselor have high interest match, followed by Library Technical Assistant and part-time Copywriter. The rest jobs do not match the user's interest well. Therefore, comparison matrix can be established, as shown in Table 3. The eigenvector is then calculated and normalized. Each element in the obtained vector represents an element value of the column of interest match (U1) in W_{65} (Table 4).

Table 3 A pairwise comparison matrix to estimate the weights of jobs with respect to interest match

	Jobs	1	2	3	4	5	6	7	8	9	10
1	**Athletic Fitness Model for Photo Shoot**	1.00	1.00	0.33	0.50	0.50	0.33	0.33	1.00	1.00	1.00
2	**Data research Internship**	1.00	1.00	0.33	0.50	0.50	0.33	0.33	1.00	1.00	1.00
3	**Languages Teachers (online)**	3.00	3.00	1.00	1.50	1.50	1.00	1.00	3.00	3.00	3.00
4	**Library Technical Assistant**	2.00	2.00	0.67	1.00	1.00	0.67	0.67	2.00	2.00	2.00
5	**Part-time Copywriter**	2.00	2.00	0.67	1.00	1.00	0.67	0.67	2.00	2.00	2.00
6	**Student Brand Ambassador**	3.00	3.00	1.00	1.50	1.50	1.00	1.00	3.00	3.00	3.00
7	**Summer Resident Camp Counselor**	3.00	3.00	1.00	1.50	1.50	1.00	1.00	3.00	3.00	3.00
8	**Telephone Survey Advisor**	1.00	1.00	0.33	0.50	0.50	0.33	0.33	1.00	1.00	1.00
9	**Temp Retail**	1.00	1.00	0.33	0.50	0.50	0.33	0.33	1.00	1.00	1.00
10	**Warehouse Operatives**	1.00	1.00	0.33	0.50	0.50	0.33	0.33	1.00	1.00	1.00

Table 4 Normalized weights of jobs with respect to interest match by considering user's situation/preferences

Jobs	eigenvector	weights (normalized)

Athletic Fitness Model for Photo Shoot	-0.16	0.06
Data research Internship	-0.16	0.06
Languages Teachers (online)	-0.48	0.17
Library Technical Assistant	-0.32	0.11
Part-time Copywriter	-0.32	0.11
Student Brand Ambassador	-0.48	0.17
Summer Resident Camp Counselor	-0.48	0.17
Telephone Survey Advisor	-0.16	0.06
Temp Retail	-0.16	0.06
Warehouse Operatives	-0.16	0.06

4.3.4 Consistency check

Since the comparison matrix is generated by pairwise comparisons, inconsistencies may arise between the weights (Alonso & Lamata, 2006). Consistency ratio (CR) is calculated to check the consistency:

$$CR = \frac{CI}{RI}$$

where CI is the consistency index, and RI is the average value of CI for random matrices using the Saaty scale (Saaty, 1990). CI is given as:

$$CI = \frac{\lambda_{max} - n}{n - 1}$$

where λ_{max} is the maximum eigenvalue of the comparison matrix and n is the number of the rows and columns of the comparison matrix. The RI values (Table 5) related to n is given by Saaty (2005).

Table 5 Random Index values (Saaty, 2005)

n	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.52	0.89	1.11	1.25	1.35	1.40	1.45	1.49

If the comparison matrix is completely consistent, $CI = \lambda_{max} - n = 0$. As the inconsistency becomes larger the CR increases. It is recommended that CR should be less than or equal to 0.10. Inconsistency may be thought of as an indicator for further adjustment. But the adjustment should not be too large, nor too small. Small adjustments would have no consequence.

4.3.5 Cluster weights and weighted super matrix

If all the clusters are equally important it is not necessary to make cluster comparisons, and the cluster weights are set to 1/n in the cluster matrix. The value of n is equal to the number of clusters. However, the clusters in a network may not be equally important and nodes in one cluster may be connected to other nodes in another cluster. So, it needs to be compared for how important its influence is on itself compared to the other clusters it links to. To compare clusters, we take each cluster in turn as the parent and pairwise compare all the clusters it connects to for importance with respect to their influence on it. After this process, the cluster weights matrix is generated (Table 6).

Taking the first column in Table 6 for instance, it indicates that salary and benefit won't influence salary, while salary depends on work conditions, suitability and jobs, and suitability and jobs are about twice times more important for salary than work conditions. The last column of w1-w4 represent users' preferences on the cluster weights. For example, if a user prefers high salary and does not care what he/her can learn from the job, w1 might be much higher than others. We determine w1-w4 by using questionnaire to each student user. Thus, each user can have distinct w1-w4.

The values in the cluster weights matrix are used to weight the unweighted supermatrix by multiplying the value in the (cluster X, cluster Y) cell of the cluster weighted matrix with the value in each cell in the (cluster X, cluster Y) component of the unweighted supermatrix to produce the weighted supermatrix.

For an unweighted supermatrix W_{uw} with n rows and n columns. There are n clusters in the network, and W_{ij} is a sub-matrix in W_{uw} which represents the influence of cluster i on cluster j (i=1, 2, …, n; j= 1, 2, …, n). If W_{ij} is not zero matrix, each column in W_{ij} has a sum of 1, which means the columns in W_{uw} may have a sum greater than 1.

The cluster weights matrix W_{cw} corresponding to W_{uw} can be expressed as:

$$W_{cw} = \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1n} \\ v_{21} & v_{22} & \cdots & v_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ v_{n1} & v_{n2} & \dots & v_{nn} \end{bmatrix}$$

where v_{ij} is the weight of the ith cluster among the n clusters that influence on cluster j, so each column in W_{cw} has a sum of 1.

The weighted supermatrix W_w can be calculated by:

$$W_w = \begin{bmatrix} v_{11} \cdot W_{11} & v_{12} \cdot W_{12} & \cdots & v_{1n} \cdot W_{1n} \\ v_{21} \cdot W_{21} & v_{22} \cdot W_{22} & \cdots & v_{2n} \cdot W_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ v_{n1} \cdot W_{n1} & v_{n2} \cdot W_{n2} & \dots & v_{nn} \cdot W_{nn} \end{bmatrix}$$

where each column in W_w has a sum of 1.

For example, the value 0.20 (cluster W, cluster S) in Table 6 is used to multiply each of the five values in the block (cluster W, cluster S) in the unweighted supermatrix shown in Appendix A. Each column in the weighted supermatrix has a sum of 1.

Table 6 cluster weights matrix

	S	B	W	U	Jobs
S	0	0	0	0	w1
B	0	0.20	0	0	w2
W	0.20	0.10	0.20	0.30	w3
U	0.40	0.30	0.40	0.30	w4
Jobs	0.40	0.40	0.40	0.40	0

4.3.6 Limiting matrix and ranked priorities of jobs

In the ANP we look for steady state priorities from a limit super matrix. To obtain the limit we must raise the weighted supermatrix to (k+1) power. After enough iterations, the limiting matrix can be obtained. In the limiting matrix, each column of the matrix has same values. The priorities of the candidate jobs are obtained by locally normalizing the corresponding values in the appropriate columns of the limit matrix. They can be found in the areas of P in the limiting matrix (Table 2).

4.4 Experimental data

To test our mathematical model, we collected and structured 10 representative jobs from studentjob.co.uk (Appendix A). According to the requirements of our problem, we have generated 10 fictional people with different situations and preferences. Each fictional student includes the information of gender, age, living place, expected salary, expected recreation hours, interests, personality, skills, and past experiences. We define the set of possible options for these factors and use Monte Carlo method to generate random values for these factors and assigned them to the ten students. For example, personality options consist of 16 types (such as the Counselor, the Nurturer) (Myers and Myers, 1990). We generate random numbers from 1-16 and assign each student a value. The communication skill options, however, are simply divided into groups of low, medium and high and we will still use Monte Carlo to assign the generated option to each student. We follow the same approach to generate other factors that make up a student's profile. It is also worthy to note that we need to have consistency check in order to eliminate possible disagreements. It is possible that the random method will generate some contradictory data. For example, communication skill may be set to low while interest is set to, say, a sale job that requires high communication skill. For those data, we need to fix them manually.

Theoretically, we can also generate a random living place for each student and use some software such as geographic information system to calculate the distance between living place and job's workplace. We can also estimate possible commuting methods and overall commuting costs. It is feasible but complex to implement, therefore, in our experiment for simplicity we have directly derived commuting costs for student users in relation to 10 jobs working sites.

5 Results

Table 7 shows a fictional student A's opinion about the importance of criteria clusters influencing selection of a best job. According to this student's profile, we filled out questionnaire and then converted it to comparison matrix to get cluster weights (w1-w4 in Table 6) which together with between-cluster weights are used for the normalization of unweighted super matrix. Those cluster weights reflecting user's preferences thus affect the student's ranking of priorities for candidate jobs. We can see that Student A attaches great importance to improving his/her own skills from summer job. According to his/her situation/preferences, we obtain the weights of the 10 jobs with respect to six criteria (Table 8). Complete unweighted and weighted supermatrix normalized by cluster weights matrix are attached to Appendix B.

Combining Table 7 and Table 8, we can conclude that Student A has the following characters:

- He pays more attention to the improvement of his own skills and abilities (a higher weight of benefit);
- He is outgoing and enthusiastic with some teamwork skill and leadership (a high weight of teamwork jobs); and
- He has some experience in student work.

Table 7 Student A's opinion on the importance of criteria clusters with respect to best job selection

Cluster	Salary	Benefit	Work conditions	Suitability
Weight	0.200	0.400	0.200	0.200

Table 8 Weights of job criteria considering Student A's preferences

Jobs	skills related to interests	Commuting	interest match	skill match	personality match	experience match
Athletic Fitness Model for Photo Shoot	0.167	0.045	0.056	0.143	0.083	0.105
Data research Internship	0.056	0.136	0.056	0.048	0.125	0.158
Languages Teachers (online)	0.111	0.136	0.167	0.143	0.083	0.158
Library Technical Assistant	0.056	0.045	0.111	0.143	0.083	0.105
Part-time Copywriter	0.111	0.136	0.111	0.048	0.083	0.105
Student Brand Ambassador	0.167	0.136	0.167	0.095	0.125	0.105
Summer Resident Camp Counselor	0.167	0.091	0.167	0.143	0.125	0.158
Telephone Survey Advisor	0.056	0.136	0.056	0.095	0.125	0.053
Temp Retail	0.056	0.045	0.056	0.143	0.125	0.053
Warehouse Operatives	0.056	0.091	0.056	0.000	0.042	0.000

After consistency check and 80-100 iterations, we can get Student A's limiting matrix (Appendix C), all columns of super matrix reach the same point. In this case, the weights in the P region (Table 2) of supermatrix is the final job priorities. The results after local normalization are shown in the column P1 of Table 9. The higher priority a job scores, the better it is for Student A. The best job for this student is Brand Ambassador with a rate of 0.127. Languages Teachers (online) and Summer Resident Camp Counselor are also good choices for Student A due to computed high priorities. Considering Student A's personality, hobbies and past experience, the best job found by the model meets our expectation.

We follow the same procedure to solve job priorities for the other nine students. The results are listed in Table 9. According to the randomly generated person attributes, the following profiles can be summarized for the total 10 fictional students (Pn represents Student n）:

P1 is Student A, outgoing, emulous and with past experience in teaching student. P2 is athletic and has good physical fitness but poor academic performance, wants to earn money through summer job. P3 is introverted, careful with good computer technology and hopes to work remotely at home and have more free time in the summer vacation. P4 has a little teaching experience, wants to improve leadership and communication skills through summer job and values work safety and working environment. P5 is outgoing, enthusiastic, and fond of outdoor activities with strong leadership, hopes to get improved through summer job. P6 has relatively poor technical skills and physical fitness, but good communication skills and sales experience, and pays more attention to salary. P7 is introverted but careful with some teaching assistant work experience and cares about the working environment, hopes to improve study ability. P8 has good physical fitness, but lacks of professional skills, hopes to improve leadership, and gain some friendship. P9 is patient and careful, hopes to find work related to the school with teachers and fellow students, and pays more attention to the suitability of work than benefit. P10 is outgoing and good at communication, he/she pays more attention to salary.

Rows in Table 9 correspond to 10 jobs, and columns correspond to 10 fictional persons. The grids in red indicate high priority jobs for the persons corresponding the columns. The best jobs solved by the model agree well with the users' situation/preferences. By contrast, green grids indicate lowest priority jobs that should be never accepted by the users considered. Those jobs are in obvious contradiction with the personality and interests of the users.

Table 9 Normalized priorities of candidate jobs for ten fictional people. Color spectrum from red to green indicates high to low priorities.

Jobs	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
Athletic Fitness Model for Photo Shoot	0.098	0.142	0.077	0.069	0.104	0.086	0.077	0.094	0.085	0.106
Data research Internship	0.093	0.072	0.153	0.124	0.087	0.077	0.092	0.084	0.098	0.068
Languages Teachers (online)	0.126	0.099	0.116	0.145	0.113	0.102	0.128	0.111	0.118	0.096
Library Technical Assistant	0.107	0.107	0.126	0.115	0.092	0.107	0.124	0.091	0.132	0.110
Part-time Copywriter	0.088	0.083	0.133	0.132	0.068	0.079	0.149	0.095	0.134	0.086
Student Brand Ambassador	0.127	0.093	0.089	0.099	0.136	0.122	0.099	0.118	0.109	0.109
Summer Resident Camp Counselor	0.119	0.078	0.067	0.081	0.163	0.086	0.076	0.120	0.078	0.104
Telephone Survey Advisor	0.098	0.104	0.085	0.081	0.072	0.138	0.090	0.095	0.092	0.126
Temp Retail	0.090	0.111	0.086	0.087	0.075	0.139	0.090	0.094	0.081	0.105
Warehouse Operatives	0.055	0.111	0.070	0.067	0.089	0.064	0.074	0.099	0.072	0.090

6 Sensitivity Analysis

6.1 The influence of the cluster weights matrix

Cluster weights matrix comprises inter-cluster weights as well as weights (w1-w4 in Table 6) in relation to user's preferences. The cluster weights, w1-w4, are obtained by questionnaire so they are usually quite arbitrarily determined. We try to explore the sensitivity of cluster weights with an example of Student B (fictional person 2). Student B is athletic and has good physical fitness. The reason for looking for a summer job is to raise money, so in the cluster weights matrix, we set the weights that the Jobs cluster depends on others are 0.5 for salary (w1), 0.1 for benefit (w2), 0.2 for both work conditions (w3) and suitability (w4). Thus, there is no doubt that the best job for him/her is Athletic Fitness Model for Photo Shoot (Table 10), which offers a relative high salary and suitable for his physical condition. Now, let’s assume that he/she has modified the goal. He/she wants to improve the leadership and teamwork skill rather than make money. The weights of salary and benefit should be switched (0.1 for salary and 0.5 for benefit). Then the job priorities (the 3rd column in Table 10) are totally different from the previous job priorities by pursuing high paid jobs. The best job for he/she is found to be Student Brand Ambassador and Summer Resident Camp Counselor is also a good choice. These two jobs meet the goal of improving leadership and teamwork skills. It’s worth mentioning that with the adjusted pursuit the priorities of the jobs that are related to self-improvement have all increased.

The Cluster Weights matrix not only reflects the influence among clusters, but also exerts the influence of user preferences on the result normalization through W1-W4 (Table 5). W1-W4 is the most arbitrary weight in the whole super matrix, which is obtained by the user's survey questions. And we take Student B (Fictional person 2) best Job selection as an example, and discuss the influence of cluster weights on the decision results.

Table 10 Impacts of modified cluster weights on the job priorities for Students B and C

Jobs	Student B Original	Student B modified	Student C Original	Student C modified
Athletic Fitness Model for Photo Shoot	0.142	0.085	0.094	0.117
Data research Internship	0.072	0.088	0.084	0.082
Languages Teachers (online)	0.099	0.105	0.111	0.097
Library Technical Assistant	0.107	0.087	0.091	0.091
Part-time Copywriter	0.083	0.092	0.095	0.097
Student Brand Ambassador	0.093	0.129	0.118	0.106
Summer Resident Camp Counselor	0.078	0.120	0.120	0.104
Telephone Survey Advisor	0.104	0.105	0.095	0.103
Temp Retail	0.111	0.093	0.094	0.098
Warehouse Operatives	0.111	0.096	0.099	0.105

Besides, changes in between-cluster weights in the cluster weights matrix may also lead to different results. The 4th and 5th columns in Table 10 show the changes in resulting job priorities for the fictional person 8 (Student C) if the between-cluster weights are altered. This student has good physical fitness and hopes to improve leadership and gain some friendship, but lacks of professional skills. The best job for him/her is Summer Resident Camp Counselor, which meets his/her preference of improving leadership and gaining friendship. This result is under the condition that the influences of benefit, work conditions, suitability, and jobs clusters on the benefit cluster are 0.2, 0.1, 0.3, and 0.4, respectively (Table 6). If we assume that the influence of suitability on benefit is much greater than other clusters, which means the benefit we get from the job depends on whether we are suitable for the job. We set 0.1, 0.1, 0.6, and 0.2 as the weights to reflect this assumption. The resulting job priorities are changed as shown in the rightmost column of Table 10. The best job now turns to be Athletic Fitness Model for Photo Shoot, which is suitable for his/her good physical fitness. The jobs that may match his/her preference of improving leadership and gaining friendships are not the best choices because the student does not have professional skills given the new assumption that the suitability cluster mainly decides the benefit.

6.2 The influence of nodes in supermatrix

The nodes in W_{i6} and W_{6j} (i=2, 3, 4; j=2, 3, 4) reflect the associations and feedbacks between individual jobs and criteria nodes. The nodes in W_{56} and W_{65}, however, take into account both individual user information and job situation. In Table 9, each row corresponds to a unique job, the priorities in the same row for different students considerably differ from each other, suggesting users' distinct situations exert large influences on the resulting job ranking. It implies that the proposed model is relatively sensitive to user situation and preferences. Persons 2 (P2) and 10 (P10) share same personality (outgoing) and purpose (earning money). The sole difference is that P2 has good physical fitness and P10 is good at communication. The prioritized jobs for them are different. P2's best summer job is Athletic Fitness Model for Photo Shoot and P10's is Telephone Survey Advice. Other nodes in sub-matrices (W42, W52, W33, W43, W53, W44, W54, W45, and W55) represent the inter-criteria relationship. They are basically independent of individual users and jobs, and reflect general understanding of the criteria. Making change on those nodes will also impact the results but we recommend keeping them unchanged throughout the process.

The nodes in Wi6 and W6j (I =2, 3, 4; J =2, 3, 4) reflects the contact and feedback between individual jobs and criteria nodes. The nodes in W56 and W65 took individual user and individual job into consideration. Each row in Table 9 shows that for the same job, different user situations have a significant impact on the priorities of jobs.

7 Presentation

There are 14 billion mobile devices worldwide in 2020[1]. There are millions of apps on the Apples store and Google Play. Many young people have mobile phones. Developing a mobile App capable of running this proposed solution will be very helpful for high school students who want to seek appropriate summer jobs. Figure 3 show some screenshots of this App. We need to install database over cloud for storing job data collected from online

[1] https://www.statista.com/statistics/245501/multiple-mobile-device-ownership-worldwide/

job websites and reprocessed for use in the ANP model. Necessary scripts should be developed to automatically grab and process raw job data. The App can access the remote job database. The model is also programmed and deployed over cloud. The App is responsible for providing interface to receive user information and running the remote model which requires job data and user data.

Figure 3a is a greeting page showing the name of this App. Then comes with Figure 3b and c, which are wizard pages guiding the user to enter necessary information that required by the model. The user is asked to provide age, gender, personalities, skills, expected salary, location, interests, and working experience. User's location can be obtained by GPS on the mobile. The location information is required to compute commuting cost criteria in the model. After that, the user is led to a questionnaire page (Figure 3d) where the user's opinion is requested on the importance of four criteria clusters (salary, other benefits, work conditions and job suitability) when choosing a favorable job. When data collection is completed, the App can run to find best job for the user. The user also has the option to start or cancel running the model. There is also option for users to browse all candidate jobs. If the model finishes running, a page as shown in Figure 3e appear where the most appropriate jobs are displayed. By clicking those best jobs, the user can move to view the details of the jobs. If it is the right job, the user should pick up phone right now to call the numbers appearing on the job detail page.

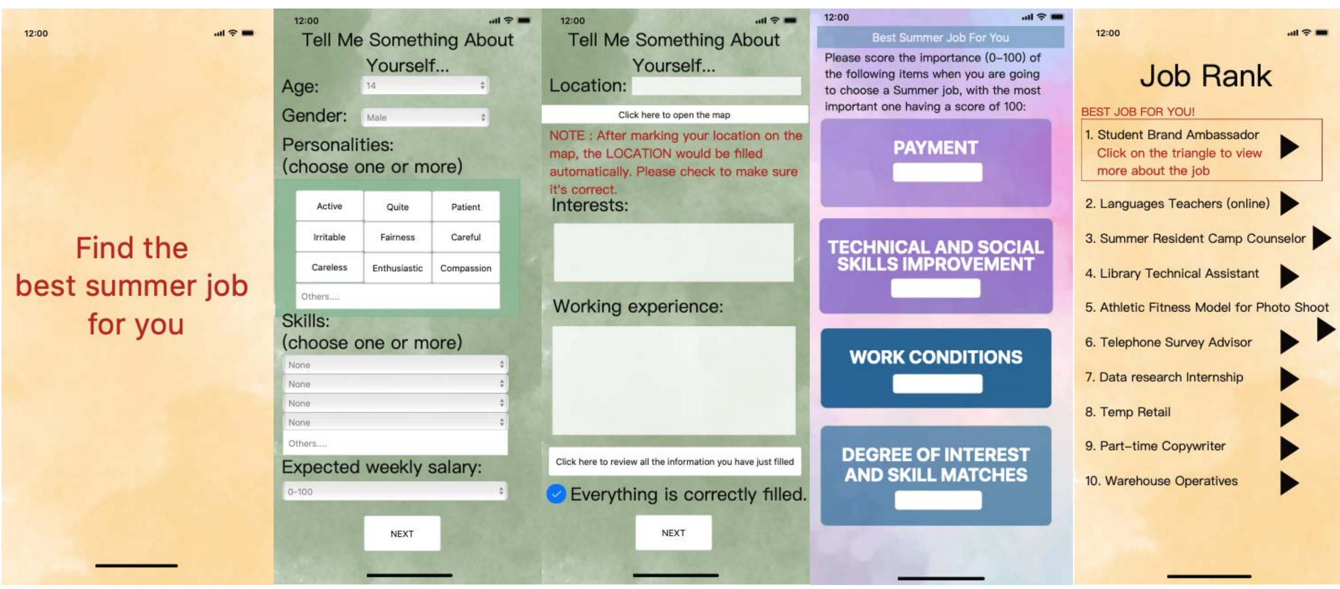

Figure 3 Screenshots of mobile Best Summer Job App. a)-e) from left to right, top down. a) Greeting page; b) and c) User information collection; d) Questionnaire; e) Resulting job ranks.

8 Strengths and Limitations

8.1 Strengths

1) We develop ANP model to solve best job selection problem. In our model, four criteria clusters: salary, skill and social benefits, work conditions, and suitability,

containing total 15 criteria are considered. The best job is scored highest. Not only do we get the best Job, we can actually get all jobs sorted by priorities, thus giving users more jobs options.

2) The best job selection is made by fully considering job characteristics. Some jobs (like internship) offer little salary, but students can benefit a lot from knowledge and skills. Some jobs may offer more salary but need more physical activities. Those kind of job characteristics impact the ranking through connecting alternative nodes (job nodes) with criteria nodes in ANP.

3) Student's Situation/Preferences have been fully considered. This is realized in three aspects. First, the four criteria clusters can have different weights (w1-w4 in Table 6), which reflects users' opinion on evaluating the goodness of a job. Some users may concern about higher pay, while others may care about improvement of technical and social skill. Second, user's situation affects relative weights of some criteria. For example, the commuting weights of different jobs vary with the users' residence. Third, the skills, their personalities, and interests that students possess will affect the suitability of a job for themselves, as represented in the super matrix where job and user information jointly determine.

4) The established model has full extendibility. We can add more criteria as criteria nodes in ANP if we need, since ANP provides such capability. It is theoretically possible to support decisions on unlimited jobs if computing resources allow. Also, unlimited users are supported. Each user can create an ANP supermatrix to find the best and most suitable job from candidate jobs.

8.2 Limitations

1) When there are too many criteria implemented in the model, the supermatrix will become very large. Because the solution requires tens of iterations, it may require large computing resources to complete the computation.

2) As inter-criteria weights are computed through comparison matrix with relative importance measured by Saaty's scale, they are subject to objectiveness. Therefore, for some criteria being able to be determined quantitatively such as salary for each job, we prefer to using their quantities to solve jobs' relative importance with respect to salary.

9 Conclusion

We established an analytic network process model to solve best summer job selection problem. In our solution, summer jobs are evaluated from total 15 criteria in four aspects: salaries the jobs can offer, technical and social skills the students can learn from the jobs, work conditions, and job suitability. Both job characteristics and user situation/preferences are fully considered in the model. Therefore, this model can recommend best jobs in highest agreement with the users' situation/preferences.

The high school student users are required to provide a few information and let us know their preferences on choosing job, and the established solution can do the rest, including creating ANP supermatrix, computing weight matrices and solving all jobs' priorities. Not only the best job, which has a highest score of priority, is recommended, but all candidate jobs are ranked according to their computed priorities, providing more job options for users. A mobile App can be developed to present this solution to high school students as potential users. We present some screenshots of such App and briefly introduce its functionality.

10 Citations and References

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Appendices

Appendix A Structured job data

Job	Location	Hourly rate ($)	Weekly salary ($)	Leadership/ Teamwork	Friend-ship	Technical skills	Communication skills	Workplace environment	Working hours per week	Flexibility	Commuting	Safety	Personality requirement	Experience requirement
Athletic Model	outdoor	150	450	N/A	N/A	N/A	N/A	outdoor	3	high	depending	medium	fitness athleticism	N/A
Data research Internship	remote	0	0	participating	N/A	computer data collection	medium	N/A	25	high	depending	high	patience carefulness	data collection
Languages Teachers (online)	remote	12	240	managing	teachers students	teaching skill	good	N/A	20	low	depending	high	patience responsibility	teaching experience
Library Technical Assistant	library	16	640	N/A	staff students	arrangement skill	medium	library	40	low	depending	high	carefulness	N/A
Part-time Copywriter	remote	10	100	N/A	N/A	good command of English	N/A	N/A	10	high	depending	high	N/A	N/A
Student Brand Ambassador	school	10	200	managing representing	staff students	teaching skill sale skill	good	N/A	20	middle	depending	medium	outgoing self-motivated	sales experience
Summer Resident Camp Counselor	outdoor	N/A	270	leading managing	counselors students	management skill	good	outdoor	all week	low	depending	low	mature fun creative	working with children or youth
Telephone Survey Advisor	remote	18	540	N/A	N/A	N/A	good	N/A	30	medium	depending	high	enthusiasm	N/A
Temp Retail	urban store	18	450	N/A	N/A	sale skill	good	a store	25	low	depending	medium	N/A	N/A
Warehouse Operatives	rural warehouse	20	500	participating	employees	heavy lifting picking and packing	low	warehouse	25	low	depending	low	carefulness	N/A

Appendix B Unweighted supermatrix of best job selection by using ANP

			S	B					W					U				Jobs									
		Best job	**S**	**B1**	**B2**	**B3**	**B4**	**B5**	**W1**	**W2**	**W3**	**W4**	**W5**	**U1**	**U2**	**U3**	**U4**	**job 1**	**job 2**	**job 3**	**job 4**	**job 5**	**job 6**	**job 7**	**job 8**	**job 9**	**job 10**
	Best job	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S	**S**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
B	**B1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.50	0.13	0.07	0.53	0.06	0.03	0.27	0.21	0.42
	B2	0	0	0	0	0	0.50	0.40	0	0	0	0	0	0	0	0	0	0	0.10	0.35	0.36	0.13	0.28	0.32	0.53	0.42	0.21
	B3	0	0	0	0.20	0	0	0.20	0	0	0	0	0	0	0	0	0	0	0.20	0.09	0.07	0.20	0.11	0.06	0.07	0.05	0.05
	B4	0	0	0	0.40	0	0	0.40	0	0	0	0	0	0	0	0	0	0	0.10	0.26	0.29	0.07	0.28	0.32	0.07	0.11	0.16
	B5	0	0	0	0.40	0	0.50	0	0	0	0	0	0	0	0	0	0	0	0.10	0.17	0.21	0.07	0.28	0.26	0.07	0.21	0.16
W	**W1**	0	0.10	0.40	0.50	0.33	0.20	0.40	0	0	0.20	0.50	0.67	0.40	0.50	0.20	0.40	0.04	0.20	0.29	0.36	0.29	0.36	0.25	0.22	0.36	0.21
	W2	0	0.40	0.40	0.50	0.33	0.60	0.40	0	0	0.40	0.25	0.33	0.20	0.25	0.40	0.20	0.37	0.20	0.06	0.09	0.06	0.18	0.25	0.17	0.18	0.26
	W3	0	0.10	0	0	0.33	0.20	0.20	0	0.50	0	0	0	0.40	0	0.40	0	0.37	0.20	0.06	0.09	0.06	0.09	0.25	0.17	0.09	0.11
	W4	0	0.10	0.20	0	0	0	0	0.50	0	0	0	0	0	0.25	0	0.40	0.15	0.20	0.29	0.36	0.29	0.27	0.13	0.22	0.27	0.21
	W5	0	0.30	0	0	0	0	0	0.50	0.50	0.40	0.25	0	0	0	0	0	0.07	0.20	0.29	0.09	0.29	0.09	0.13	0.22	0.09	0.21
U	**U1**	0	0.10	0.40	0.10	0.20	0.10	0.40	0.25	0.50	0.80	0	0	0	0.40	0.50	0.40	0.14	0.10	0.14	0.13	0.14	0.29	0.31	0.20	0.17	0.11
	U2	0	0.40	0.40	0.10	0.40	0.10	0.10	0.25	0	0	0.40	0.60	0.20	0	0	0.40	0.57	0.50	0.29	0.25	0.29	0.21	0.13	0.40	0.33	0.56
	U3	0	0.30	0.20	0.80	0.20	0.80	0.40	0.25	0.50	0.20	0.20	0.20	0.60	0.20	0	0.20	0.14	0.20	0.29	0.38	0.29	0.29	0.31	0.20	0.33	0.22
	U4	0	0.20	0	0	0.20	0	0.10	0.25	0	0	0.40	0.20	0.20	0.40	0.50	0	0.14	0.20	0.29	0.25	0.29	0.21	0.25	0.20	0.17	0.11
Jobs	**job 1**	0	0.15	0	0	0.17	0	0	0.07	0.18	0.20	0.09	0.05	0.06	0.14	0.08	0.11	0	0	0	0	0	0	0	0	0	0
	job 2	0	0	0.21	0.10	0.06	0.08	0	0.14	0.10	0.16	0.12	0.14	0.06	0.05	0.13	0.16	0	0	0	0	0	0	0	0	0	0
	job 3	0	0.08	0.10	0.16	0.11	0.23	0.08	0.14	0.12	0.08	0.12	0.14	0.17	0.14	0.08	0.16	0	0	0	0	0	0	0	0	0	0
	job 4	0	0.20	0.10	0.10	0.06	0	0.17	0.09	0.05	0.08	0.11	0.05	0.11	0.14	0.08	0.11	0	0	0	0	0	0	0	0	0	0
	job 5	0	0.03	0.08	0	0.11	0	0	0.14	0.15	0.16	0.12	0.14	0.11	0.05	0.08	0.11	0	0	0	0	0	0	0	0	0	0
	job 6	0	0.06	0.16	0.16	0.17	0.23	0.25	0.14	0.12	0.12	0.09	0.14	0.17	0.10	0.13	0.11	0	0	0	0	0	0	0	0	0	0
	job 7	0	0.09	0.10	0.16	0.17	0.31	0.33	0.05	0	0	0.07	0.09	0.17	0.14	0.13	0.16	0	0	0	0	0	0	0	0	0	0
	job 8	0	0.17	0	0.11	0.06	0	0	0.14	0.08	0.12	0.12	0.14	0.06	0.10	0.13	0.05	0	0	0	0	0	0	0	0	0	0
	job 9	0	0.14	0.08	0.16	0.06	0	0	0.07	0.10	0.04	0.09	0.05	0.06	0.14	0.13	0.05	0	0	0	0	0	0	0	0	0	0
	job 10	0	0.08	0.17	0.05	0.06	0.15	0.17	0.02	0.10	0.04	0.07	0.09	0.06	0	0.04	0	0	0	0	0	0	0	0	0	0	0

Appendix C Limiting supermatrix of best job selection by using ANP

			S	B					W					U				Jobs									
		Best job	**S**	**B1**	**B2**	**B3**	**B4**	**B5**	**W1**	**W2**	**W3**	**W4**	**W5**	**U1**	**U2**	**U3**	**U4**	**job 1**	**job 2**	**job 3**	**job 4**	**job 5**	**job 6**	**job 7**	**job 8**	**job 9**	**job 10**
	Best job	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
S	**S**	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050
B	**B1**	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008
	B2	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017
	B3	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006
	B4	0.014	0.014	0.014	0.014	0.014	0.014	0.014	0.014	0.014	0.014	0.014	0.014	0.014	0.014	0.014	0.014	0.014	0.014	0.014	0.014	0.014	0.014	0.014	0.014	0.014	0.014
	B5	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013
W	**W1**	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037
	W2	0.031	0.031	0.031	0.031	0.031	0.031	0.031	0.031	0.031	0.031	0.031	0.031	0.031	0.031	0.031	0.031	0.031	0.031	0.031	0.031	0.031	0.031	0.031	0.031	0.031	0.031
	W3	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021
	W4	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020
	W5	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017
U	**U1**	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.052
	U2	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048
	U3	0.058	0.058	0.058	0.058	0.058	0.058	0.058	0.058	0.058	0.058	0.058	0.058	0.058	0.058	0.058	0.058	0.058	0.058	0.058	0.058	0.058	0.058	0.058	0.058	0.058	0.058
	U4	0.042	0.042	0.042	0.042	0.042	0.042	0.042	0.042	0.042	0.042	0.042	0.042	0.042	0.042	0.042	0.042	0.042	0.042	0.042	0.042	0.042	0.042	0.042	0.042	0.042	0.042
Jobs	**job 1**	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017
	job 2	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016
	job 3	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022
	job 4	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019
	job 5	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015
	job 6	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022
	job 7	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021
	job 8	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017
	job 9	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016
	job 10	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010